Practice Booklet

MYP 4 · Year 10 Mathematics (Extended)

Unit 10.2 · Exponents and Indices

Name: _____

10.2.1	Index laws · Questions 1–20
10.2.1b	Negative and fractional indices · Questions 21–38
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10.2.4 (b)	Multiplying and collecting surds · Questions 96–114
10.2.4b	Rationalising the denominator · Questions 115–132
10.2.5	Sequences with surds · Questions 133–151
Homework	Homework page and checking record
Answers	For checking and correcting in green pen



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How to use this booklet and your homework

READ THIS FIRST

Choosing your level

Every part of the unit has four levels. Start where you feel comfortable and move between them freely. Most questions ask you to explain as well as answer, because being able to say *why* is what makes a method stick.

Learning
The standard case with small numbers, with the method set out for you.

Practising
The method on its own, with your reasoning written out.

Challenging
Harder, with several steps, and finding what is wrong with someone else's working.

Mastery
Explaining why a rule must be true, and testing it on new cases.

Working on your homework

Pens and setting out

- Answers in blue or black non-erasable pen, or in pencil.
- Colour and highlighters are for annotating your work and showing the steps of a method.
- Never erase a mistake. Rework it underneath so you can see what changed.

Green pen is for checking. When you finish, use the answers at the back to check your work.

- **Got it wrong?** Circle the question number, then make your correction and write your error analysis in green pen.
- **Still can't work out how to solve it?** Put a star ★ next to the question number, so we know to go over it in class.

Each sheet

- Your name goes on every sheet, because they are handed in one at a time.
- Each question is worth up to 3 marks. If a question has parts (a, b, c...), each part is marked out of 3. At the end of each level, write your score out of the total for that level.
- Questions are numbered 1, 2, 3... all the way through the booklet, so we can say exactly which ones to do.
- If you are stuck after trying, ask for help. Showing me what you tried is a good start.

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10.2.1 Index laws

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
base	the number being multiplied by itself	2 in 2^3
index	how many times the base is used as a factor	3 in 2^3
power	a base with an index	$2^3 = 8$
same base	the same number or letter	5^2 and 5^4

Laws (same base only): $a^m \times a^n = a^{m+n}$ · $a^m \div a^n = a^{m-n}$ · $(a^m)^n = a^{mn}$ · $a^0 = 1$

Learning

1 Write $2 \times 2 \times 2 \times 2$ as a power

2 a) Write $2^3 \times 2^2$ in full, with every 2 showing b) Count the 2s and write the answer as one power of 2

3 Write $3^4 \times 3^5$ as a single power of 3

4 Write $7^8 \div 7^3$ as a single power of 7

5 Write $(5^2)^3$ in full, then as a single power of 5

6 Work out a) 9^0 b) 100^0 c) 1^0

Learning score _____ / 27

Practising

7 Simplify a) $x^4 \times x^3$ b) $y^9 \div y^2$ c) $(a^3)^5$

8 Simplify $(2x^3)^4$

9 Simplify $\dfrac{12x^5}{3x^2}$

10 Simplify $3a^2b \times 4ab^3$

11 Simplify $(3a^2b)^3$

Practising score _____ / 21

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10.2.1 Index laws

CHALLENGING · MASTERY

Name:

Challenging

12 Simplify $\frac{(2x^3)^2 \times 3x^4}{6x^5}$

13 Work out $4^0 + (3x)^0 + 5x^0$, where x is not zero

14 Find n: a) $3^n \div 3^4 = 3^6$ b) $(5^n)^2 = 5^{12}$

15 Find and describe the mistake. Sam writes $2^3 \times 3^2 = 6^5$

16 Write $2^3 \times 2^3 \times 2^3$ as a power of 2 in two different ways

Challenging score _____ / 18

Mastery

17 Use the divide law on $5^3 \div 5^3$ to explain why $5^0 = 1$

18 Explain why $3^4 \times 3^2 = 3^6$, but $3^4 + 3^2$ is not 3^6. Work out both sides to check

19 Show that $(2^3)^4 = (2^4)^3$, then explain why $(a^m)^n = (a^n)^m$

20 A square has side 2^5 cm. Write its area as a power of 2 and work it out

Mastery score _____ / 12

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10.2.1b Negative and fractional indices

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
reciprocal	a fraction turned upside down	the reciprocal of 4 is $\frac{1}{4}$
negative index	means take the reciprocal	$a^{-n} = \frac{1}{a^n}$
fractional index	means a root; the denominator is the root	$a^{\frac{m}{n}}$ = (n-th root of a)^m
root	undoes a power	the cube root of 8 is 2

Do the root first, then the power.

Learning

21 Continue the pattern. $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$, $2^{-1} =$ ___, $2^{-2} =$ ___

22 Write as a fraction a) 2^{-3} b) 10^{-2} c) 5^{-1}

23 Work out a) $25^{\frac{1}{2}}$ b) $100^{\frac{1}{2}}$ c) $8^{\frac{1}{3}}$ d) $27^{\frac{1}{3}}$

24 Write the reciprocal of a) 4 b) $\frac{2}{3}$

Learning score _____ **/ 30**

Practising

25 Evaluate a) 3^{-2} b) 4^{-3} c) 10^{-4}

26 Evaluate a) $(\frac{1}{2})^{-4}$ b) $(\frac{2}{3})^{-2}$

27 Evaluate a) $16^{\frac{1}{4}}$ b) $32^{\frac{1}{5}}$ c) $1000^{\frac{1}{3}}$

28 Evaluate a) $8^{\frac{2}{3}}$ b) $27^{\frac{2}{3}}$ c) $4^{\frac{3}{2}}$

29 Write $3^{-2} \times 3^5$ as a single power of 3, then evaluate it

Practising score _____ **/ 36**

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10.2.1b Negative and fractional indices

CHALLENGING · MASTERY

Name:

Challenging

30 Evaluate a) $9^{-\frac{1}{2}}$ b) $27^{-\frac{2}{3}}$ c) $16^{-\frac{3}{4}}$

31 Evaluate $(\frac{4}{9})^{-\frac{3}{2}}$

32 Simplify $x^{-2} \times x^{5} \div x^{-1}$

33 Find and describe the mistake. Jo says $2^{-3} = -8$

34 a) Write $\frac{1}{32}$ as a power of 2 b) Write 8×2^{-5} as a single power of 2, then as a fraction

Challenging score _____ **/ 24**

Mastery

35 Use $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ to explain why $a^{\frac{1}{2}}$ must be the square root of a

36 A student says $4^{\frac{3}{2}} = 6$ because $4 \times 3 \div 2 = 6$. Explain the mistake and give the correct value

37 Evaluate $8^{-\frac{2}{3}}$, saying which rule you use at each step

38 Put in order, smallest first, without a calculator: 2^{-1}, $4^{\frac{1}{2}}$, 8^{0}, $27^{-\frac{1}{3}}$

Mastery score _____ **/ 12**

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10.2.2 Comparing exponents

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
common base	the same base on both sides	4 and 8 are both powers of 2
equate	make two things equal	$2^x = 2^5$, so x = 5

Steps: 1 choose a common base. 2 rewrite each side as a power of it. 3 equate the exponents. 4 solve and check.

Learning

39 Solve $2^x = 8$

40 Solve a) $5^x = 25$ b) $3^x = 81$ c) $10^x = 1000$

41 Write as a power of 2 a) 4 b) 16 c) 32

42 Solve $2^x = 64$ by writing 64 as a power of 2

43 Solve $5^x = 1$

Learning score _____ / 27

Practising

44 Solve a) $2^x = 256$ b) $5^x = 625$ c) $10^x = 1\,000\,000$

45 Solve $25^x = 125$

46 Solve $4^x = \frac{1}{16}$

47 Solve $8^x = 16$

48 Solve $9^x = 27$

Practising score _____ / 21

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10.2.2 Comparing exponents

CHALLENGING · MASTERY

Name:

Challenging

49 Solve a) $27^x = 9$ b) $3^x = \frac{1}{81}$ c) $8^x = \frac{1}{4}$

50 Solve $4^x = 32$

51 Solve $(\frac{1}{2})^x = 8$

52 Find and describe the mistake. Dee solves $9^x = 27$. She writes $3^x = 3^3$, so x = 3

53 Solve $2^x = 0.25$

Challenging score _____ / 21

Mastery

54 Explain why $3^x = 5$ cannot be solved by writing both sides as a power of 3. Between which two whole numbers is x?

55 Explain why $1^x = 1$ has many solutions, so we cannot compare exponents when the base is 1

56 a) A culture of bacteria starts with 1 cell and doubles every hour. After how many hours are there 1024 cells? b) Another culture triples every hour. After how many hours does it reach 729 cells?

57 Solve $16^x = 8$ and $8^y = 16$. Show that $x \times y = 1$ and explain why

Mastery score _____ / 15

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10.2.3 Algebraic exponents

LEARNING · PRACTISING

Name:

Key vocabulary and steps

Word	Meaning
exponent	the power; here it contains x
common base	write both sides as powers of the same base
equate	make the exponents equal, then solve

Steps: 1 match the bases. 2 use the index laws on the exponents. 3 equate the exponents and solve. 4 check by substitution.

Learning

58 Write 8 as a power of 2

59 Simplify a) $2^3 \times 2^4$ b) $(2^3)^2$

60 Write as a power of 2 a) 4^x b) 8^x c) 16^x

61 Solve $2^{x+1} = 2^5$

62 Solve $3^{2x} = 3^8$

Learning score _____ / 24

Practising

63 Solve $9^x = 3^{x+4}$

64 Solve $5^{x+1} = 25^x$

65 Solve $2^{x-1} = 4^{x+1}$

66 Solve $3^{2x+1} = 27^{x-1}$

67 Solve $5^{x+2} = 25^{x-3}$

Practising score _____ / 15

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10.2.3 Algebraic exponents

Name:

Challenging

68 Solve $2^{3x-1} = 4^{x+2}$

69 Solve $4^{x+1} = 8^{x}$

70 Solve $(\frac{1}{2})^{x} = 4^{x+3}$

71 Find and describe the mistake. Ali solves $4^{x+1} = 2^{x+5}$. He writes $4^{x+1} = 2^{2x+1}$

72 Check by substitution that x = 3 is a solution of $4^{x+1} = 2^{x+5}$

Challenging score _____ / 15

Mastery

73 Solve $2^{x} \times 4^{x+1} = 128$

74 Explain why $27^{x-1} = 3^{3x-3}$ and not 3^{3x-1}

75 a) A population is 5×2^{t} after t hours. When is it 160? b) When is 5×2^{t} equal to $5 \times 4^{t-2}$?

76 Write an equation of the form $3^{ax+b} = 9^{cx+d}$ whose solution is x = 2. Show that it works

Mastery score _____ / 15

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10.2.4 (a) Surds: what they are and simplifying

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
surd	a root that cannot be written exactly as a whole number or fraction	$\sqrt{7}$
irrational	a number whose decimal never ends or repeats	$\sqrt{2}$
perfect square	a whole number squared	1, 4, 9, 16, 25, 36, 49
highest perfect square factor	the largest square that divides the number	for 72 it is 36
simplest form	the number under the root has no square factor	$6\sqrt{2}$

Rule: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. Pull out the highest perfect square factor.

Learning

77 List the first ten perfect squares

78 Which of these are surds? $\sqrt{16}$ $\sqrt{7}$ $\sqrt{25}$ $\sqrt{10}$

79 Complete: $\sqrt{18} = \sqrt{9} \times \sqrt{2} =$ ___ $\sqrt{2}$

80 Simplify $\sqrt{8}$ by writing it as $\sqrt{4} \times \sqrt{2}$

81 Simplify $\sqrt{12}$. Hint: which perfect square goes into 12?

Learning score _____ / 15

Practising

82 Simplify $\sqrt{48}$

83 Simplify $\sqrt{75}$

84 Simplify $\sqrt{72}$, using the highest perfect square factor

85 Simplify $\sqrt{45}$

86 Simplify $5\sqrt{8}$

Practising score _____ / 15

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10.2.4 (a) Surds: what they are and simplifying

CHALLENGING · MASTERY

Name:

Challenging

87 Simplify a) $\sqrt{180}$ b) $3\sqrt{50}$ c) $\sqrt{200}$

88 Write $6\sqrt{2}$ as a single surd $\sqrt{n}$

89 Find and describe the mistake. A student writes $\sqrt{9} + \sqrt{16} = \sqrt{25} = 5$

90 Simplify $\sqrt{98}$ and $\sqrt{32}$

91 Simplify $\frac{\sqrt{75}}{5}$

Challenging score _____ / 21

Mastery

92 A student simplifies $\sqrt{72}$ as $2\sqrt{18}$. Is the value right? Is it in simplest form? Explain

93 Show that $\sqrt{8}$ and $\sqrt{18}$ are both whole-number multiples of $\sqrt{2}$

94 Explain why $\sqrt{2}$ cannot be written exactly as a fraction or a finite decimal

95 A square has area 50 cm^2. Find the exact side length in simplest form

Mastery score _____ / 12

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10.2.4 (b) Multiplying and collecting surds

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
like surds	surds with the same number under the root	$3\sqrt{2}$ and $5\sqrt{2}$
unlike surds	different numbers under the root, so they cannot be collected	$\sqrt{2}$ and $\sqrt{3}$

Rules: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ · $\sqrt{a} \times \sqrt{a} = a$ · collect like surds · expand with the box method, then simplify

Learning

96 Work out $\sqrt{2} \times \sqrt{3}$

97 Work out $\sqrt{5} \times \sqrt{5}$

98 Work out $3\sqrt{2} \times 2\sqrt{5}$

99 Simplify $4\sqrt{3} + 2\sqrt{3}$

100 Simplify $7\sqrt{5} - 3\sqrt{5}$

Learning score _____ / 15

Practising

101 Simplify $3\sqrt{2} \times 4\sqrt{6}$

102 Expand $\sqrt{3}(5 - \sqrt{3})$

103 Expand $(3 + \sqrt{2})(1 + \sqrt{2})$

104 Simplify $\sqrt{48} - \sqrt{12}$

105 Simplify $\sqrt{8} + \sqrt{50}$

Practising score _____ / 15

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10.2.4 (b) Multiplying and collecting surds

CHALLENGING · MASTERY

Name:

Challenging

106 Expand $(3 + \sqrt{2})^2$

107 Expand $(4 + \sqrt{3})(4 - \sqrt{3})$

108 Expand $(\sqrt{5} + 2)(\sqrt{5} - 2)$

109 Find and describe the mistake. Tom writes $(3 + \sqrt{2})^2 = 9 + 2 = 11$

110 Simplify $\sqrt{45} + \sqrt{20} - \sqrt{5}$

Challenging score _____ **/ 15**

Mastery

111 A student writes $\sqrt{3} + \sqrt{2} = \sqrt{5}$. Use a calculator to show this is wrong, and explain why

112 Find the exact area of a square with side $(2 + \sqrt{3})$ cm

113 A right-angled triangle has shorter sides $2\sqrt{3}$ cm and $\sqrt{6}$ cm. Find the hypotenuse exactly

114 Find x if $\sqrt{x} + \sqrt{4x} = 3\sqrt{5}$

Mastery score _____ **/ 12**

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10.2.4b Rationalising the denominator

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
rationalise	rewrite so the denominator is a whole number	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
conjugate	same terms, opposite sign between them	$3 + \sqrt{2}$ and $3 - \sqrt{2}$
difference of two squares	$(a + b)(a - b) = a^2 - b^2$	$(3 + \sqrt{2})(3 - \sqrt{2}) = 7$

Multiply top and bottom by $\sqrt{b}$, or by the conjugate of the denominator.

Learning

115 Work out $\sqrt{7} \times \sqrt{7}$

116 Work out $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ and write the answer as a fraction

117 Rationalise a) $\frac{3}{\sqrt{3}}$ b) $\frac{10}{\sqrt{5}}$

118 Write the conjugate of a) $3 + \sqrt{2}$ b) $4 - \sqrt{5}$ c) $\sqrt{7} + 1$

Learning score _____ **/ 21**

Practising

119 Rationalise a) $\frac{6}{\sqrt{3}}$ b) $\frac{4}{\sqrt{2}}$ c) $\frac{15}{\sqrt{5}}$

120 Rationalise $\frac{3}{2\sqrt{3}}$

121 Rationalise $\frac{10}{3\sqrt{2}}$

122 Expand $(4 + \sqrt{3})(4 - \sqrt{3})$

123 Rationalise $\frac{5}{3 + \sqrt{2}}$

Practising score _____ **/ 21**

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10.2.4b Rationalising the denominator

CHALLENGING · MASTERY

Name:

Challenging

124 Rationalise $\frac{1}{\sqrt{2}+1}$

125 Rationalise $\frac{2}{3-\sqrt{5}}$

126 Rationalise $\frac{6}{\sqrt{7}+1}$

127 Find and describe the mistake. Ben rationalises $\frac{1}{2+\sqrt{3}}$ by multiplying the top and bottom by $2+\sqrt{3}$

128 Rationalise $\frac{4}{\sqrt{5}-1}$

Challenging score _____ / 15

Mastery

129 Explain why multiplying a fraction by its conjugate over itself does not change its value, even though it looks different

130 Rationalise $\frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}}$

131 Simplify $\frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}}$

132 A rectangle has area 6 cm^2 and length $(3+\sqrt{3})$ cm. Find the width in the form $a-\sqrt{b}$

Mastery score _____ / 12

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10.2.5 Sequences with surds

LEARNING · PRACTISING

Name:

Key vocabulary

Word	Meaning	Example
common difference	d: the gap between terms in an arithmetic sequence	$\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$: d = $\sqrt{3}$
common ratio	r: what you multiply by each time in a geometric sequence	1, $\sqrt{5}$, 5: r = $\sqrt{5}$
general term	u_n: the rule for any term	$u_n = n\sqrt{3}$

Arithmetic: $u_n = u_1 + (n - 1)d$. Geometric: $u_n = u_1 \times r^{n-1}$. Simplify every surd first.

Learning

133 Simplify a) $\sqrt{12}$ b) $\sqrt{27}$

134 Simplify $\sqrt{20}$, $\sqrt{45}$ and $\sqrt{80}$

135 Write the next term: $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, ...

136 a) Simplify each term of $\sqrt{3}$, $\sqrt{12}$, $\sqrt{27}$ b) Find the common difference d

137 Write u_n for $\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$, ...

Learning score _____ **/ 21**

Practising

138 For $\sqrt{8}$, $\sqrt{32}$, $\sqrt{72}$: simplify each term, then find d and u_n

139 The sequence 2, $2\sqrt{3}$, 6, $6\sqrt{3}$ is geometric. Find r and the next term

140 For the geometric sequence 1, $\sqrt{5}$, 5, $5\sqrt{5}$, ... find r and u_6

141 The sequence $\sqrt{5}$, $3\sqrt{5}$, $5\sqrt{5}$, ... is arithmetic. Find d, u_n and u_{10}

142 Is $\sqrt{2}$, 2, $2\sqrt{2}$, 4 arithmetic or geometric? Give d or r

Practising score _____ **/ 15**

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10.2.5 Sequences with surds

CHALLENGING · MASTERY

Name:

Challenging

143 Find u_n and u_{20} for $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, ...

144 A geometric sequence has $u_1 = 3$ and $u_2 = 3\sqrt{2}$. Find r, u_3 and u_5

145 An arithmetic sequence has $u_1 = 2\sqrt{3}$ and $d = \sqrt{3}$. Find u_n and show that $u_8 = \sqrt{243}$

146 Find and describe the mistake. Maya says $\sqrt{3}$, $\sqrt{12}$, $\sqrt{27}$ is geometric with $r = \sqrt{4} = 2$, because $12 \div 3 = 4$

147 Show that the ratio of 4, $2\sqrt{2}$, 2, ... is $\frac{\sqrt{2}}{2}$

Challenging score _____ / 15

Mastery

148 Explain why simplifying each surd first makes it easier to tell whether a sequence is arithmetic

149 a) Write the first three terms of an arithmetic sequence with $u_1 = \sqrt{2}$ and $d = 3\sqrt{2}$ b) Write u_n

150 A geometric sequence has $u_2 = 6$ and $u_4 = 12$ with r positive. Find r and u_1 in simplest form

151 $\sqrt{2}$, $\sqrt{6}$, $\sqrt{18}$, ... Show it is geometric, then find u_{10} in simplest form

Mastery score _____ / 15

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Homework and checking record

HAND THIS PAGE IN WITH YOUR SHEETS

Name:

Your teacher will tell you which sheet and questions to do. Choose your level, set out your work in pen or pencil, then check it with the answers at the back using a **green pen**: circle wrong question numbers, and star ★ any you still cannot solve.

Sheet	Questions	Which ones I did	Checked in green ✔	Teacher
10.2.1 Index laws	Q1–20			
10.2.1b Negative and fractional indices	Q21–38			
10.2.2 Comparing exponents	Q39–57			
10.2.3 Algebraic exponents	Q58–76			
10.2.4 (a) Surds: what they are and simplifying	Q77–95			
10.2.4 (b) Multiplying and collecting surds	Q96–114			
10.2.4b Rationalising the denominator	Q115–132			
10.2.5 Sequences with surds	Q133–151			

Error analysis

Circle the number of any question you got wrong. Star ★ it if you still cannot solve it after correcting. Then write the questions you need to rework, and what you will do differently:

One thing I understand better now

Score: Parent / guardian signature:

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Answers

CHECK IN GREEN PEN

10.2.1 Index laws

Learning	**1**	2^4
	2	a) $(2 \times 2 \times 2) \times (2 \times 2)$ b) Five 2s, so $2^5 = 32$
	3	$3^{4+5} = 3^9$
	4	$7^{8-3} = 7^5$
	5	$(5^2)^3 = 5^2 \times 5^2 \times 5^2 = 5^6$
	6	a) 1 b) 1 c) 1. Any number to the power 0 is 1
Practising	**7**	a) x^7 b) y^7 c) a^{15}
	8	$2^4 \times x^{12} = 16x^{12}$
	9	$12 \div 3 = 4$ and $x^{5-2} = x^3$, so $4x^3$
	10	$3 \times 4 = 12$, $a^2 \times a = a^3$, $b \times b^3 = b^4$, so $12a^3b^4$
	11	$3^3 \times (a^2)^3 \times b^3 = 27a^6b^3$
Challenging	**12**	$(2x^3)^2 = 4x^6$, so the top is $12x^{10}$. Then $12x^{10} \div 6x^5 = 2x^5$
	13	$1 + 1 + 5 \times 1 = 7$. The index 0 only applies to the x in $5x^0$
	14	a) $n - 4 = 6$, so $n = 10$ b) $2n = 12$, so $n = 6$
	15	The law only works when the bases are the same, and the indices are added only for the same base. $2^3 \times 3^2 = 8 \times 9 = 72$ (but $6^5 = 7776$)
	16	2^9, or $(2^3)^3 = 2^9$. Both are 512
Mastery	**17**	$5^3 \div 5^3 = 5^{3-3} = 5^0$. But any number divided by itself is 1, so $5^0 = 1$
	18	Multiplying adds the indices: $3^4 \times 3^2 = 81 \times 9 = 729 = 3^6$. Adding the values gives $81 + 9 = 90$, which is not 729. There is no law for adding powers
	19	Both are 2^{12}. The index is $m \times n$, and $m \times n = n \times m$
	20	Area = $(2^5)^2 = 2^{10} = 1024$ cm^2

10.2.1b Negative and fractional indices

Learning

21 Each step down divides by 2, so $2^{-1} = \frac{1}{2}$ and $2^{-2} = \frac{1}{4}$

22 a) $\frac{1}{2^3} = \frac{1}{8}$ b) $\frac{1}{100}$ c) $\frac{1}{5}$

23 a) The square root of 25 is 5 b) 10 c) The cube root of 8 is 2 d) 3

24 a) $\frac{1}{4}$ b) $\frac{3}{2}$

Practising

25 a) $\frac{1}{9}$ b) $\frac{1}{64}$ c) $\frac{1}{10\,000}$

26 Flip the fraction, then use the positive index. a) $2^4 = 16$ b) $(\frac{3}{2})^2 = \frac{9}{4}$

27 a) 2 b) 2 c) 10

28 Root first, then power. a) $2^2 = 4$ b) $3^2 = 9$ c) $2^3 = 8$

29 $3^{-2+5} = 3^3 = 27$

Challenging

30 a) $\frac{1}{3}$ b) $\frac{1}{3^2} = \frac{1}{9}$ c) $\frac{1}{2^3} = \frac{1}{8}$

31 Flip the fraction to make the index positive: $(\frac{9}{4})^{\frac{3}{2}}$. The square root of $\frac{9}{4}$ is $\frac{3}{2}$, and cubed it is $\frac{27}{8}$

32 $x^{-2+5-(-1)} = x^4$

33 A negative index means reciprocal, it does not make the number negative. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

34 a) 2^{-5} b) $2^3 \times 2^{-5} = 2^{-2} = \frac{1}{4}$

Mastery

35 Add the indices: $\frac{1}{2} + \frac{1}{2} = 1$, so $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1 = a$. The number that multiplies by itself to give a is the square root of a

36 The denominator is the root and the numerator is the power. Square root of 4 is 2, then $2^3 = 8$

37 Negative index: take the reciprocal. Denominator 3: the cube root of 8 is 2. Numerator 2: $2^2 = 4$. So $8^{-\frac{2}{3}} = \frac{1}{4}$

38 The values are $\frac{1}{2}$, 2, 1 and $\frac{1}{3}$. In order: $27^{-\frac{1}{3}}$, 2^{-1}, 8^0, $4^{\frac{1}{2}}$

10.2.2 Comparing exponents

Learning

39 $8 = 2^3$, so $2^x = 2^3$ and $x = 3$

40 a) $x = 2$ b) $x = 4$ c) $x = 3$

41 a) 2^2 b) 2^4 c) 2^5

42 $64 = 2^6$, so $x = 6$

43 $1 = 5^0$, so $x = 0$

Practising

44 a) $x = 8$ b) $x = 4$ c) $x = 6$

45 $25 = 5^2$ and $125 = 5^3$, so $5^{2x} = 5^3$, $2x = 3$ and $x = \frac{3}{2}$

46 $\frac{1}{16} = 4^{-2}$, so $x = -2$

47 $8 = 2^3$, $16 = 2^4$, so $3x = 4$ and $x = \frac{4}{3}$

48 $9 = 3^2$, $27 = 3^3$, so $2x = 3$ and $x = \frac{3}{2}$

Challenging

49 a) $3^{3x} = 3^2$, $x = \frac{2}{3}$ b) $3^x = 3^{-4}$, $x = -4$ c) $2^{3x} = 2^{-2}$, $x = -\frac{2}{3}$

50 $4 = 2^2$, $32 = 2^5$, so $2x = 5$ and $x = \frac{5}{2}$

51 $\frac{1}{2} = 2^{-1}$, so $2^{-x} = 2^3$, $-x = 3$ and $x = -3$

52 She forgot that $9 = 3^2$, so $9^x = 3^{2x}$. Then $3^{2x} = 3^3$, $2x = 3$ and $x = \frac{3}{2}$

53 $0.25 = \frac{1}{4} = 2^{-2}$, so $x = -2$

Mastery

54 5 is not a whole-number power of 3. $3^1 = 3$ and $3^2 = 9$, so x is between 1 and 2

55 1 to any power is 1, so every value of x works. Comparing exponents needs a base that gives a different value for each exponent

56 a) $2^x = 1024 = 2^{10}$, so 10 hours b) $3^x = 729 = 3^6$, so 6 hours

57 $x = \frac{3}{4}$ and $y = \frac{4}{3}$, so $x \times y = 1$. The exponents are reciprocals because one equation is the other turned round

10.2.3 Algebraic exponents

Learning

58 $8 = 2^3$

59 a) 2^7 b) 2^6

60 a) $(2^2)^x = 2^{2x}$ b) 2^{3x} c) 2^{4x}

61 The bases match, so $x + 1 = 5$ and $x = 4$

62 $2x = 8$, so $x = 4$

Practising

63 $9^x = 3^{2x}$, so $2x = x + 4$ and $x = 4$

64 $25^x = 5^{2x}$, so $x + 1 = 2x$ and $x = 1$

65 $4^{x+1} = 2^{2(x+1)} = 2^{2x+2}$, so $x - 1 = 2x + 2$ and $x = -3$

66 $27^{x-1} = 3^{3(x-1)} = 3^{3x-3}$, so $2x + 1 = 3x - 3$ and $x = 4$

67 $25^{x-3} = 5^{2x-6}$, so $x + 2 = 2x - 6$ and $x = 8$

Challenging

68 $4^{x+2} = 2^{2x+4}$, so $3x - 1 = 2x + 4$ and $x = 5$

69 $2^{2x+2} = 2^{3x}$, so $2x + 2 = 3x$ and $x = 2$

70 $2^{-x} = 2^{2x+6}$, so $-x = 2x + 6$ and $x = -2$

71 The 2 from $4 = 2^2$ must multiply the whole exponent: $4^{x+1} = 2^{2(x+1)} = 2^{2x+2}$. Then $2x + 2 = x + 5$ and $x = 3$

72 Left side: $4^4 = 256$. Right side: $2^8 = 256$. They are equal, so $x = 3$ works

Mastery

73 $2^x \times 2^{2x+2} = 2^{3x+2}$. $128 = 2^7$, so $3x + 2 = 7$ and $x = \frac{5}{3}$

74 $27 = 3^3$, so $27^{x-1} = (3^3)^{x-1} = 3^{3(x-1)}$. The 3 multiplies the whole exponent $x - 1$, which gives $3x - 3$

75 a) $2^t = 32$, so $t = 5$ b) $2^t = 2^{2t-4}$, so $t = 2t - 4$ and $t = 4$

76 For example $3^{x+2} = 9^x$. Solve: $x + 2 = 2x$, so $x = 2$. Check: $3^4 = 81$ and $9^2 = 81$

10.2.4 (a) Surds: what they are and simplifying

Learning	**77**	1, 4, 9, 16, 25, 36, 49, 64, 81, 100
	78	$\sqrt{7}$ and $\sqrt{10}$. $\sqrt{16} = 4$ and $\sqrt{25} = 5$ are whole numbers, so they are not surds
	79	$\sqrt{9} = 3$, so $\sqrt{18} = 3\sqrt{2}$
	80	$2\sqrt{2}$
	81	$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
Practising	**82**	$\sqrt{16} \times \sqrt{3} = 4\sqrt{3}$
	83	$\sqrt{25} \times \sqrt{3} = 5\sqrt{3}$
	84	$\sqrt{36} \times \sqrt{2} = 6\sqrt{2}$
	85	$\sqrt{9} \times \sqrt{5} = 3\sqrt{5}$
	86	$5 \times 2\sqrt{2} = 10\sqrt{2}$
Challenging	**87**	a) $\sqrt{36} \times \sqrt{5} = 6\sqrt{5}$ b) $3 \times 5\sqrt{2} = 15\sqrt{2}$ c) $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$
	88	$6\sqrt{2} = \sqrt{36} \times \sqrt{2} = \sqrt{72}$
	89	You cannot add under the roots. $\sqrt{9} = 3$ and $\sqrt{16} = 4$, so the sum is $3 + 4 = 7$
	90	$\sqrt{98} = 7\sqrt{2}$ and $\sqrt{32} = 4\sqrt{2}$
	91	$\sqrt{75} = 5\sqrt{3}$, so the answer is $\sqrt{3}$
Mastery	**92**	The value is right, because $2\sqrt{18} = \sqrt{4}\sqrt{18} = \sqrt{72}$. It is not in simplest form: $\sqrt{18} = 3\sqrt{2}$, so the answer is $6\sqrt{2}$. We use the highest perfect square factor, 36
	93	$\sqrt{8} = 2\sqrt{2}$ and $\sqrt{18} = 3\sqrt{2}$
	94	$\sqrt{2}$ is irrational: its decimal 1.41421356... never ends and never repeats, so any decimal we write is only an approximation
	95	Side $= \sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$ cm

10.2.4 (b) Multiplying and collecting surds

Learning	**96**	$\sqrt{6}$
	97	5, because a square root times itself gives the number
	98	$3 \times 2 = 6$ and $\sqrt{2} \times \sqrt{5} = \sqrt{10}$, so $6\sqrt{10}$
	99	$6\sqrt{3}$. They are like surds, so add the numbers in front
	100	$4\sqrt{5}$
Practising	**101**	$12\sqrt{12} = 12 \times 2\sqrt{3} = 24\sqrt{3}$
	102	$5\sqrt{3} - \sqrt{3} \times \sqrt{3} = 5\sqrt{3} - 3$
	103	$3 + 3\sqrt{2} + \sqrt{2} + 2 = 5 + 4\sqrt{2}$
	104	$4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3}$
	105	$2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$
Challenging	**106**	$9 + 3\sqrt{2} + 3\sqrt{2} + 2 = 11 + 6\sqrt{2}$
	107	$16 - 3 = 13$. The middle terms cancel
	108	$5 - 4 = 1$
	109	He squared each term and left out the middle terms. $(3 + \sqrt{2})(3 + \sqrt{2}) = 9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$
	110	$3\sqrt{5} + 2\sqrt{5} - \sqrt{5} = 4\sqrt{5}$
Mastery	**111**	$\sqrt{3} + \sqrt{2} \approx 1.732 + 1.414 = 3.146$, but $\sqrt{5} \approx 2.236$. Roots do not add under one root. $\sqrt{3}$ and $\sqrt{2}$ are unlike surds, so the sum cannot be simplified
	112	$(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$ cm^2
	113	$h^2 = 12 + 6 = 18$, so $h = \sqrt{18} = 3\sqrt{2}$ cm
	114	$\sqrt{4x} = 2\sqrt{x}$, so $3\sqrt{x} = 3\sqrt{5}$, $\sqrt{x} = \sqrt{5}$ and $x = 5$

10.2.4b Rationalising the denominator

Learning

115 7

116 $\frac{\sqrt{2}}{2}$. We multiplied by 1, so the value does not change but how it is written does

117 a) $\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$ b) $\frac{10\sqrt{5}}{5} = 2\sqrt{5}$

118 a) $3 - \sqrt{2}$ b) $4 + \sqrt{5}$ c) $\sqrt{7} - 1$

Practising

119 a) $2\sqrt{3}$ b) $2\sqrt{2}$ c) $3\sqrt{5}$

120 Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$: $\frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

121 $\frac{10\sqrt{2}}{6} = \frac{5\sqrt{2}}{3}$

122 $16 - 3 = 13$, a whole number

123 Multiply by $\frac{3 - \sqrt{2}}{3 - \sqrt{2}}$. The denominator is $9 - 2 = 7$, so the answer is $\frac{15 - 5\sqrt{2}}{7}$

Challenging

124 $\frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$

125 $\frac{2(3 + \sqrt{5})}{9 - 5} = \frac{3 + \sqrt{5}}{2}$

126 $\frac{6(\sqrt{7} - 1)}{7 - 1} = \sqrt{7} - 1$

127 He used the same sign. The conjugate is $2 - \sqrt{3}$. Then $\frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$

128 $\frac{4(\sqrt{5} + 1)}{5 - 1} = \sqrt{5} + 1$

Mastery

129 The conjugate over itself is a fraction equal to 1, and multiplying by 1 does not change the value. The value stays the same but is written differently, with a rational denominator

130 $\frac{\sqrt{5}(\sqrt{5} - \sqrt{2})}{5 - 2} = \frac{5 - \sqrt{10}}{3}$

131 $\frac{2 + \sqrt{3}}{1} + \frac{2 - \sqrt{3}}{1} = 4$, because each denominator becomes $4 - 3 = 1$

132 Width $= \frac{6}{3 + \sqrt{3}} = \frac{6(3 - \sqrt{3})}{9 - 3} = 3 - \sqrt{3}$ cm

10.2.5 Sequences with surds

Learning

133 a) $2\sqrt{3}$ b) $3\sqrt{3}$

134 $2\sqrt{5}$, $3\sqrt{5}$ and $4\sqrt{5}$

135 $4\sqrt{2}$. Each term adds $\sqrt{2}$

136 a) $\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$ b) $d = \sqrt{3}$

137 $u_n = n\sqrt{3}$

Practising

138 $2\sqrt{2}$, $4\sqrt{2}$, $6\sqrt{2}$. $d = 2\sqrt{2}$ and $u_n = 2n\sqrt{2}$

139 $r = \sqrt{3}$, since $2\sqrt{3} \div 2 = \sqrt{3}$. The next term is $6\sqrt{3} \times \sqrt{3} = 18$

140 $r = \sqrt{5}$. $u_6 = (\sqrt{5})^5 = 25\sqrt{5}$

141 $d = 2\sqrt{5}$. $u_n = \sqrt{5} + (n - 1) \times 2\sqrt{5} = (2n - 1)\sqrt{5}$. $u_{10} = 19\sqrt{5}$

142 Each term is the last times $\sqrt{2}$, so it is geometric with $r = \sqrt{2}$

Challenging

143 Simplified: $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$. $u_n = n\sqrt{2}$, so $u_{20} = 20\sqrt{2}$

144 $r = \sqrt{2}$. $u_3 = 3 \times 2 = 6$. $u_5 = 3 \times (\sqrt{2})^4 = 12$

145 $u_n = 2\sqrt{3} + (n - 1)\sqrt{3} = (n + 1)\sqrt{3}$. $u_8 = 9\sqrt{3} = \sqrt{81} \times \sqrt{3} = \sqrt{243}$

146 She only checked the first pair. $\sqrt{27} \div \sqrt{12} = 3\sqrt{3} \div 2\sqrt{3} = 1.5$, not 2. The terms simplify to $\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$, so it is arithmetic with $d = \sqrt{3}$

147 $2\sqrt{2} \div 4 = \frac{\sqrt{2}}{2}$, and $2 \div 2\sqrt{2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. Both give the same ratio

Mastery

148 After simplifying, the terms are multiples of the same surd, so you can see the gaps. For example $2\sqrt{3}$, $3\sqrt{3}$, $4\sqrt{3}$ has a constant gap $\sqrt{3}$

149 a) $\sqrt{2}$, $4\sqrt{2}$, $7\sqrt{2}$ b) $u_n = \sqrt{2} + (n - 1) \times 3\sqrt{2} = (3n - 2)\sqrt{2}$

150 $u_4 = u_2 \times r^2$, so $r^2 = 2$ and $r = \sqrt{2}$. $u_1 = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

151 $r = \sqrt{3}$, since $\sqrt{6} \div \sqrt{2} = \sqrt{3}$. $u_{10} = \sqrt{2} \times (\sqrt{3})^9 = \sqrt{2} \times 81\sqrt{3} = 81\sqrt{6}$