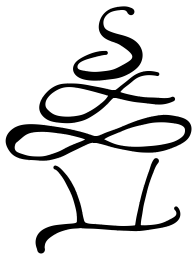
Practice Booklet

MYP 4 · Year 10 Mathematics (Extended)

Unit 10.1 · Sequences

Name: _____

10.1.1a	The border problem · Questions 1–10
10.1.1b	The painted cube · Questions 11–20
10.1.1	c The painted cube as functions · Questions 21–30
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10.1.2a	Arithmetic sequences · Questions 41–50
10.1.2b	Tiling patterns and the nth term · Questions 51–60
10.1.2	c Finding u_1 and d from two terms · Questions 61–70
10.1.3a	Geometric sequences · Questions 71–80
10.1.3b	Finding u_1 and r from two terms · Questions 81–90
10.1.3	c Arithmetic or geometric? · Questions 91–100
10.1.4	Verify and justify · Questions 101–110
Homework	Homework page and checking record
Answers	For checking and correcting in green pen



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How to use this booklet and your homework

READ THIS FIRST

Choosing your level

Every part of the unit has four levels. Start where you feel comfortable and move between them freely. Most questions ask you to explain as well as answer, because being able to say *why* is what makes a method stick.

Learning
Small numbers, with counters or a diagram beside you.

Practising
The standard question, with your reasoning written out.

Challenging
Working backwards, and finding what is wrong with someone else's thinking.

Mastery
Explaining why a rule must be true, and testing it on new cases.

Working on your homework

Pens and setting out

- Answers in blue or black non-erasable pen, or in pencil.
- Colour and highlighters are for annotating your work and showing the steps of a method.
- Never erase a mistake. Rework it underneath so you can see what changed.

Green pen is for checking. When you finish, use the answers at the back to check your work.

- **Got it wrong?** Circle the question number, then make your correction and write your error analysis in green pen.
- **Still can't work out how to solve it?** Put a star ★ next to the question number, so we know to go over it in class.

Each sheet

- Your name goes on every sheet, because they are handed in one at a time.
- Each question is worth up to 3 marks. If a question has parts (a, b, c...), each part is marked out of 3. At the end of each level, write your score out of the total for that level.
- Questions are numbered 1, 2, 3... all the way through the booklet, so we can say exactly which ones to do.
- If you are stuck after trying, ask for help. Showing me what you tried is a good start.

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10.1.1a The border problem

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
border	the squares round the outside edge of a grid	36 squares in a 10 × 10 grid
grid	rows and columns of squares	an n × n grid has n rows and n columns
expression	numbers and letters with operations, but no equals sign	4n − 4
rule	a formula that gives the answer for any size of grid	border = 4n − 4
n	a letter for the size of the grid	n = 10 for a 10 × 10 grid

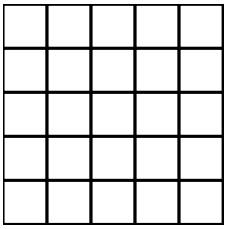
Different ways of counting give different-looking expressions. If they are all correct they must be **equal**. The border of an n × n grid is 4n − 4.

Learning

1 Here is a 5 × 5 grid.

a) Shade the border squares

b) Count them



2 Use the rule border = 4n − 4 to find the number of border squares in an 18 × 18 grid

3 Ravi counts the border of a 7 × 7 grid as 7 + 7 + 5 + 5.

a) Describe how Ravi sees the border

b) Work out his total

c) Check it with 4n − 4

Learning score _____ **/ 18**

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10.1.1a The border problem

PRACTISING

Name:

Practising

4 Nadia says the border of an 11 × 11 grid is 4 × 11 = 44.
a) Explain her mistake
b) Find the correct number

5 A grid is 14 × 14. Write a calculation using the big square take away the small square, and check it with 4n − 4

6 A square grid has 88 squares in its border. Use 4n − 4 = 88 to find the size of the grid

Practising score _____ / 12

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10.1.1a The border problem

CHALLENGING · MASTERY

Name:

Challenging

7 Show that $n + 2(n - 1) + (n - 2)$ simplifies to $4n - 4$

8 Expand and simplify $n^2 - (n - 2)^2$ to show that it is also $4n - 4$

Challenging score _____ / 6

Mastery

9 A rectangle grid is m squares wide and n squares tall. Show that the border has $2m + 2n - 4$ squares, then check with a 10×6 grid

10 A square grid has 121 squares inside its border. How many squares are in the border?

Mastery score _____ / 6

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10.1.1b The painted cube

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
cube	a solid with 6 square faces, all the same size	a dice
unit cube	a cube with side length 1	1 × 1 × 1
face	a flat surface of a solid. A cube has 6	one side of a dice
edge	where two faces meet. A cube has 12	a line of the cube
vertex (corner)	where edges meet. A cube has 8	a corner
nth term	a rule that gives any term from its position n	12(n − 2)

For a cube with side n (n of 2 or more): 3 painted faces 8, 2 painted faces $12(n-2)$, 1 painted face $6(n-2)^2$, no paint $(n-2)^3$.

Learning

11 A cube with side 8 cm is painted red on every face, then cut into 1 cm cubes. a) How many 1 cm cubes are there in total? b) How many have 3 painted faces? c) How many have no paint?

12 A 6 × 6 × 6 cube is painted on every face and cut into unit cubes. a) How many have exactly 1 painted face? b) How many have exactly 2 painted faces?

13 A 7 × 7 × 7 cube is painted and cut into unit cubes. Complete: no paint $(n-2)^3$ = ___^{3} = ___, 1 face $6(n-2)^2$ = ___, 2 faces $12(n-2)$ = ___, 3 faces = ___. Check that the four counts add to 7^3

Learning score _____ **/ 18**

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10.1.1b The painted cube

PRACTISING

Name:

Practising

14 A 10 × 10 × 10 cube is painted and cut into unit cubes. a) How many have exactly 2 painted faces? b) How many have exactly 1 painted face? c) Show that the counts for 3, 2, 1 and 0 painted faces add to 1000

15 A big cube is painted on every face and cut into unit cubes. Exactly 216 of the unit cubes have no paint. What is the side length of the big cube?

16 Write the number of unit cubes with exactly 2 painted faces in an n × n × n cube. Use your rule to find the number when n = 15

Practising score _____ / 15

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10.1.1b The painted cube

CHALLENGING · MASTERY

Name:

Challenging

17 The number of unit cubes with 1 painted face is $6(n-2)^2$. a) Expand and simplify $6(n-2)^2$. b) Show that your answer gives 24 when $n = 4$

18 In a painted cube, exactly 294 unit cubes have 1 painted face. Find the side length of the cube, then find how many unit cubes have no paint

Challenging score _____ / 9

Mastery

19 Show that the four counts always add up to n^3: expand and simplify $(n-2)^3 + 6(n-2)^2 + 12(n-2) + 8$

20 A cuboid $3 \times 4 \times 5$ is painted on every face and cut into unit cubes. Find how many unit cubes have 3, 2, 1 and 0 painted faces. Check that they add to $3 \times 4 \times 5$

Mastery score _____ / 6

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10.1.1c The painted cube as functions

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
linear	a rule where n has power 1; its graph is a straight line	12(n − 2)
quadratic	a rule where the highest power of n is 2; its graph is a parabola	$6(n-2)^2$
cubic	a rule where the highest power of n is 3; its graph is a steeper curve	$(n-2)^3$
constant	a rule that never changes; its graph is a flat line	8
function	a rule that gives one output for each input	n = 5 gives 54
intersection	a point where two graphs cross; both rules give the same value	(4, 24)
domain	the set of input values that are allowed	n = 2, 3, 4, …
growth	how quickly the output gets bigger as the input gets bigger	cubic grows fastest

Painted cube of side n (n ≥ 2): 0 painted faces $(n-2)^3$, 1 painted face $6(n-2)^2$, 2 painted faces 12(n − 2), 3 painted faces 8.

Learning

21 A 5 × 5 × 5 cube is painted on every face and cut into 125 small cubes. How many small cubes have a) 3 painted faces b) 0 painted faces?

22 Use 12(n − 2) to find the number of edge cubes (2 painted faces) in a) a 3 × 3 × 3 cube b) a 6 × 6 × 6 cube

23 Complete for a 6 × 6 × 6 cube: the number of small cubes with 0, 1, 2 and 3 painted faces. Check that they add up to 216.

Learning score _____ **/ 15**

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10.1.1c The painted cube as functions

PRACTISING

Name:

Practising

24 Name the type of function (linear, quadratic or cubic) and describe its graph: a) $6(n-2)^2$ b) $(n-2)^3$ c) $12(n-2)$

25 A cube has side n = 9. Find the number of small cubes with 0, 1, 2 and 3 painted faces. Check the total is 729.

26 A cube has 108 small cubes with exactly 2 painted faces. Find n.

Practising score _____ **/ 15**

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10.1.1c The painted cube as functions

Name:

Challenging

27 A cube has 150 small cubes with exactly 1 painted face. Find n, then find the number of inside cubes.

28 Find the smallest whole number n for which there are more inside cubes than edge cubes. Show a table of values to support your answer.

Challenging score _____ / 6

Mastery

29 Explain why the painted cube formulae only work for whole numbers n ≥ 2, and why a graph of them should show separate points, not a joined line.

30 A student says: “The cubic always gives the most small cubes because it has the highest power.” Use values of n to show when this is true and when it is not.

Mastery score _____ / 6

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10.1.1d What is a sequence?

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
sequence	a list of numbers that follows a rule	2, 5, 8, 11, ...
term	each number in a sequence	11 is a term
position	where a term sits: 1st, 2nd, 3rd, ...	11 is in position 4
recursive formula	a term-to-term rule written with the first term	$u_1 = 2$, $u_n = u_{n-1} + 3$
explicit formula	a formula that gives the term from its position n	$u_n = 3n - 1$

u_n is the term in position n. The small number is the position, not a multiplier.

Learning

31 For the sequence 6, 11, 16, 21, 26, ... write down a) the 1st term b) the 4th term c) u_3

32 Write the next two terms of each sequence. a) 5, 9, 13, 17, ... b) 90, 81, 72, 63, ... c) 3, 6, 12, 24, ...

33 Write the first five terms of the sequence with first term 12 and the term-to-term rule "subtract 5"

Learning score _____ / 21

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10.1.1d What is a sequence?

PRACTISING

Name:

Practising

34 Write the first five terms of each sequence. a) $u_n = 3n + 4$ b) $u_n = 10 - 2n$

35 A sequence has $u_1 = 4$ and $u_n = u_{n-1} + 7$. Write u_2, u_3, u_4 and u_5

36 A sequence has the rule $u_n = 5n - 2$. a) Find u_8 b) Which position has the term 43? Show your working

Practising score _____ / 15

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10.1.1d What is a sequence?

CHALLENGING · MASTERY

Name:

Challenging

37 Sam says 3, 6, 9, … continues 12, 15, 18, … Mia says it continues 3, 6, 9, 3, 6, 9, … Can both be right? Explain, then give a third way it could continue

38 Say whether each rule is recursive or explicit, then find u_5. a) $u_n = n^2 + 1$ b) $u_1 = 2$ and $u_n = 3u_{n-1} - 1$ c) $u_n = 20 - 3n$

Challenging score _____ / 12

Mastery

39 A sequence has $u_1 = 2$, $u_2 = 3$ and $u_n = u_{n-1} + u_{n-2}$. Write the first eight terms

40 Explain the difference between a recursive formula and an explicit formula. Which is quicker for finding u_{50}, and why?

Mastery score _____ / 6

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10.1.2a Arithmetic sequences

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
arithmetic sequence	a sequence where the same amount is added each time	3, 7, 11, 15, ...
term	each number in a sequence	11 is a term
common difference, d	the amount added to get the next term, which can be negative	d = 4 for 3, 7, 11, 15
recursive formula	a rule that gives each term from the term before, with u_1	$u_n = u_{n-1} + 4$, $u_1 = 3$
explicit formula	a formula that gives any term from its position n	$u_n = 4n - 1$

To find d, take one term and subtract the previous term. The explicit formula is $u_n = u_1 + (n - 1)d$. Check any formula by substituting n = 1 and n = 2.

Learning

41 Decide whether each sequence is arithmetic. If it is, write down d

a) 4, 9, 14, 19, ...

b) 25, 21, 17, 13, ...

c) 2, 4, 8, 16, ...

d) 1, 3, 6, 10, ...

42 Write the first four terms of each sequence

a) $u_1 = 6$ and $d = 3$

b) $u_1 = 20$ and $d = -5$

c) $u_1 = 1$ and $u_n = u_{n-1} + 7$

43 The explicit formula of a sequence is $u_n = 6n + 1$

a) Find u_1, u_2 and u_3

b) Find u_{10}

Learning score _____ **/ 27**

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10.1.2a Arithmetic sequences

PRACTISING

Name:

Practising

44 Look at the sequence 9, 13, 17, 21, ...

a) Write down u_1 and d

b) Write the explicit formula $u_n = u_1 + (n - 1)d$ and simplify it

c) Find u_{30}

45 Look at the sequence 50, 44, 38, 32, ...

a) Write the recursive formula

b) Write the explicit formula

c) Find u_{15}

46 The sequence 2, 5, 8, 11, ... has $u_n = 3n - 1$

a) Is 100 a term of the sequence

b) Is 101 a term of the sequence

Explain each answer

Practising score _____ **/ 24**

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Name:

Challenging

47 A pattern of tiles has 5 tiles in pattern 1, 8 tiles in pattern 2 and 11 tiles in pattern 3. Each pattern has 3 more tiles than the one before

a) Write a formula for the number of tiles in pattern n

b) How many tiles are in pattern 40

c) Which pattern has 98 tiles

48 An arithmetic sequence has $u_3 = 14$ and $d = -3$

a) Find u_1

b) Find u_{10}

Challenging score _____ **/ 15**

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10.1.2a Arithmetic sequences

MASTERY

Name:

Mastery

49 A sequence has the explicit formula $u_n = 5n^2$

a) Write the first three terms

b) Show that the sequence is not arithmetic

50 An arithmetic sequence has $u_n = 4n - 3$

a) Write u_{n+1} in terms of n

b) Show that $u_{n+1} - u_n$ is always 4

c) What does this tell you about d

Mastery score _____ **/ 15**

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10.1.2b Tiling patterns and the nth term

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
sequence	a list of numbers that follow a rule	1, 5, 9, 13, ...
term	one number in a sequence	the 3rd term is 9
position n	where a term sits in the sequence	n = 4 is the 4th term
nth term u_n	a rule that gives the term in any position n	$u_n = 4n - 3$
common difference d	what you add to one term to get the next term, it can be negative	d = 4 or d = −2
0th term	the term that would come before the 1st term, the y-intercept on a graph	$u_0 = 1 - 4 = -3$
verify	show a result is correct by using different means	$u_5 = 4(5) - 3 = 17$

To find the nth term: find the common difference d, write the multiples of d, then find what to add or subtract. The rule is $u_n = dn + c$, where c is the 0th term. Always verify by substituting, then write a concluding statement.

Learning

51 A pattern of tiles has 3 tiles in pattern 1, 7 tiles in pattern 2 and 11 tiles in pattern 3.

a) Find the common difference

b) How many tiles will pattern 4 have?

c) Verify your answer to part b by drawing or describing the pattern

52 Complete the table for the sequence 5, 8, 11, 14, 17

Position n: 1, 2, 3, 4, 5

Multiples of 3: ___ ___ ___ ___ ___

Adjust: ___

Term u_n: 5, 8, 11, 14, 17

Write the rule for u_n

53 Find the rule u_n for the sequence 12, 22, 32, 42, ...

Then find u_{15}

Learning score _____ **/ 15**

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10.1.2b Tiling patterns and the nth term

PRACTISING

Name:

Practising

54 Find the rule u_n for the sequence 18, 15, 12, 9, ...
Then find u_{10}

55 A tile pattern has 4, 9, 14 and 19 tiles in patterns 1, 2, 3 and 4.
a) Write a rule for the number of tiles in pattern n
b) How many tiles are in pattern 40?

56 Amir says the rule for 1, 7, 13, 19, ... is $u_n = 6n - 5$
Verify his rule by substitution
Write a concluding statement

Practising score _____ / 12

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10.1.2b Tiling patterns and the nth term

CHALLENGING · MASTERY

Name:

Challenging

57 The nth term of a sequence is $u_n = 3n + 5$

a) Is 83 a term of the sequence?

b) Is 100 a term of the sequence?

Show your working

58 The points (1, 9), (2, 7) and (3, 5) show the first three terms of a sequence on a graph.

a) What is the gradient?

b) What is the y-intercept?

c) Write the rule u_n

Challenging score _____ / 15

Mastery

59 Pattern n is a rectangle of tiles n tall and n + 2 wide. Pattern 1 has 3 tiles, pattern 2 has 8, pattern 3 has 15 and pattern 4 has 24.

a) Find the first differences. Is the sequence arithmetic?

b) Find the second differences

c) Write a rule for pattern n and find pattern 10

60 The nth term of a sequence is $u_n = 5n - 12$

a) Find the 0th term

b) Which is the first positive term?

c) Explain why the graph of u_n against n should be points, not a continuous line

Mastery score _____ / 18

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10.1.2c Finding u_1 and d from two terms

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
term	one number in a sequence, named by its position	$u_3 = 11$ is the 3rd term
first term u_1	the term in position 1	$u_1 = 3$
common difference d	the amount added each time; it can be negative	$d = 4$ or $d = -4$
jump	one step of d from one term to the next	u_3 to u_7 is 4 jumps

The number of jumps between two terms is the **difference of their positions**. The term in position n is $u_1 + (n - 1)d$.

Learning

61 An arithmetic sequence has $d = 5$. How many jumps are there from u_2 to u_6?

62 An arithmetic sequence has $u_2 = 7$ and $u_5 = 19$.
a) How many jumps from u_2 to u_5?
b) Find d

63 Use $u_1 = 2$ and $d = 6$ to find u_4 and u_9.

Learning score _____ / 12

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10.1.2c Finding u_1 and d from two terms

PRACTISING

Name:

Practising

64 $u_2 = 10$ and $u_6 = 30$. Find d and u_1

65 $u_3 = 4$ and $u_7 = -12$. Find d and u_1

66 $u_4 = 23$ and $u_{10} = 59$. Find u_1, d and u_{15}

Practising score _____ **/ 9**

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10.1.2c Finding u_1 and d from two terms

CHALLENGING · MASTERY

Name:

Challenging

67 The sequence 6, 11, 16, … is arithmetic. a) Which term is 106? b) Is 150 a term? Explain

68 $u_5 = 8$ and $u_{12} = -20$. Find u_1 and d, then write the formula for u_n. Verify with u_8

Challenging score _____ / 9

Mastery

69 An arithmetic sequence has $u_2 + u_6 = 34$ and $u_9 = 41$. Find u_1 and d

70 The 10th term of an arithmetic sequence is three times the 4th term and $u_7 = 21$. Find u_1 and d

Mastery score _____ / 6

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10.1.3a Geometric sequences

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
geometric sequence	a sequence where each term is the one before multiplied by the same number	3, 6, 12, 24 …
common ratio, r	the number you multiply by: divide a term by the one before it	$r = 12 \div 6 = 2$
recursive formula	a rule that gives each term from the term before, with u_1 stated	$u_n = u_{n-1} \times 2$, $u_1 = 3$
explicit formula	a rule that gives any term from its position n	$u_n = 3 \times 2^{n-1}$
power	the small raised number: how many times to multiply	$2^4 = 2 \times 2 \times 2 \times 2 = 16$

Geometric sequences multiply by r. Arithmetic sequences add d. The explicit rule is $\mathbf{u_n = u_1 \times r^{n-1}}$: from u_1 there are n − 1 steps of × r.

Learning

71 Write the next three terms

a) 4, 12, 36, …

b) 500, 100, 20, …

c) 2, −4, 8, …

72 Find the common ratio r

a) 7, 14, 28, 56

b) 90, 30, 10

c) 4, −12, 36

d) 6, 9, 13.5

73 Is each sequence arithmetic, geometric or neither? Give d or r

a) 5, 10, 15, 20

b) 5, 10, 20, 40

c) 1, 4, 9, 16

d) 80, 40, 20, 10

Learning score _____ **/ 33**

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10.1.3a Geometric sequences

PRACTISING

Name:

Practising

74 A sequence has $u_1 = 5$ and $r = 3$

a) Write the first four terms

b) Write the recursive formula

75 A geometric sequence has $u_1 = 4$ and $r = 2$. Use $u_n = u_1 \times r^{n-1}$ to find

a) u_6 b) u_8

76 Find the 7th term of each sequence

a) 3, 6, 12, ...

b) 729, 243, 81, ...

Practising score _____ **/ 15**

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10.1.3a Geometric sequences

CHALLENGING · MASTERY

Name:

Challenging

77 A geometric sequence has $u_1 = 2$ and $u_4 = 54$

a) Find r

b) Find u_6

78 A geometric sequence has $u_2 = 20$ and $u_4 = 80$

Find the two possible values of r, and the u_1 that goes with each

Challenging score _____ / 9

Mastery

79 £500 is put in a savings account. It grows by 4% each year

a) Show that the multiplier is r = 1.04

b) The amount after 1 year is u_1 = £520. Find the amount after 3 years, to the nearest penny

80 Sequence A is 5, 10, 15, 20, ... (arithmetic) and sequence B is 5, 10, 20, 40, ... (geometric)

a) Find the first term where B is bigger than A

b) Explain why B will always stay ahead after that

Mastery score _____ / 12

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10.1.3b Finding u_1 and r from two terms

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
common ratio r	the number you multiply by to get from one term to the next	8, 12, 18, 27 has r = 1.5
first term u_1	the term in position 1	$u_1 = 2$
explicit formula	a rule that gives any term from its position n	$u_n = u_1 \times r^{n-1}$
root	the undo of a power	$\sqrt[3]{27} = 3$ because $3^3 = 27$
verify	show evidence that an answer is correct using a different method	substitute into the formula

To find u_1 and r from two terms: count the steps, **divide** the later term by the earlier term, take the **root** to find r, then divide back to u_1. An even number of steps can give two values of r.

Learning

81 A geometric sequence has r = 3 and $u_1 = 4$.
Write the first four terms

82 A geometric sequence has $u_2 = 8$ and $u_3 = 40$.
Find r

83 How many steps are there from u_3 to u_7?

Learning score _____ **/ 9**

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10.1.3b Finding u_1 and r from two terms

PRACTISING

Name:

Practising

84 A geometric sequence has $u_1 = 2$ and $u_4 = 54$.
Find r

85 A geometric sequence has $u_2 = 7$ and $u_5 = 56$.
Find r and u_1

86 A geometric sequence has $u_3 = 20$ and $u_6 = 540$.
Find r, u_1 and u_7

Practising score _____ **/ 9**

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10.1.3b Finding u_1 and r from two terms

CHALLENGING · MASTERY

Name:

Challenging

87 A geometric sequence has $u_2 = 18$ and $u_4 = 2$.
Find both possible values of r and the matching value of u_1

88 A geometric sequence has $u_2 = 6$ and $u_5 = 48$.
Find the first term that is greater than 500

Challenging score _____ / 6

Mastery

89 The numbers 5, x and 45 are consecutive terms of a geometric sequence.
Find the possible values of x

90 A geometric sequence has $u_3 = 16$ and $u_7 = 1296$.
Find both possible sequences. Write u_1, u_2, u_3 for each and verify one of them

Mastery score _____ / 6

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10.1.3c Arithmetic or geometric?

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
arithmetic sequence	add the same amount every time	4, 9, 14, 19, ...
common difference d	the amount added; subtract a term from the next	$d = 9 - 4 = 5$
geometric sequence	multiply by the same amount every time	3, 6, 12, 24, ...
common ratio r	the amount multiplied by; divide a term by the one before	$r = 6 \div 3 = 2$
explicit formula	gives any term from n	$u_n = 3 \times 2^{n-1}$
recursive formula	gives a term from the one before, plus u_1	$u_n = u_{n-1} \times 2$, $u_1 = 3$

Arithmetic sequences show **linear growth** (points on a line). Geometric sequences show **exponential growth** (points on a curve).

Learning

91 Write **arithmetic**, **geometric** or **neither** for each sequence.

a) 6, 11, 16, 21, ...

b) 5, 10, 20, 40, ...

c) 1, 2, 4, 7, ...

92 Find the next two terms of each sequence.

a) 7, 14, 28, 56, ...

b) 90, 83, 76, 69, ...

93 Write the explicit formula for the arithmetic sequence 5, 8, 11, 14, ... in the form $u_n = u_1 + (n - 1)d$

Learning score _____ / 18

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10.1.3c Arithmetic or geometric?

PRACTISING

Name:

Practising

94 A geometric sequence starts 3, 12, 48, 192, ...

a) Write the common ratio r

b) Write the explicit formula

c) Find the 6th term

95 Write the recursive formula, including the first term, for each sequence.

a) 20, 17, 14, 11, ...

b) 2, 10, 50, 250, ...

96 Plan A pays £6 on day 1 and then £6 more each day. Plan B pays 2p on day 1 and doubles each day.

a) Copy and complete the table of pay for days 1 to 6

b) Which plan has the greater pay on day 6?

c) Which type of growth does each plan show?

Practising score _____ **/ 24**

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10.1.3c Arithmetic or geometric?

CHALLENGING · MASTERY

Name:

Challenging

97 Plan B above keeps doubling. Find its pay on day 10 and on day 12, in pounds. Which day is the first when Plan B pays more than Plan A?

98 A geometric sequence has $u_1 = 5$ and $r = 3$. Write u_2, u_3 and u_4. Then explain why the differences between terms are not equal

Challenging score _____ / 6

Mastery

99 A virus is modelled by 12 cases on day 0, and the number of cases doubles every 4 days.
a) Find the number of cases on days 4, 8 and 12
b) How many cases after 40 days?
c) A second model adds 12 cases every 4 days. How many cases after 40 days? Compare the two

100 The sequence 2, 6, 18, 54, ... is geometric. Show that $u_n = 2 \times 3^{n-1}$ and find the first term that is greater than 1 000 000

Mastery score _____ / 12

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10.1.4 Verify and justify

LEARNING

Name:

Key vocabulary

Word	Meaning	Example
verify	provide evidence that validates the result	substitute n = 4, then check against a drawing
justify	give valid reasons or evidence to support an answer or conclusion	explain why the 4 and the − 3 are in 4n − 3
explain	give a detailed account including reasons or causes	say how and why a pattern grows
variable	a letter that stands for a number that can change	n is the pattern number
notation	the symbols we use, and what each one means	u_n means the nth term
concluding statement	a final sentence that says what you have shown	so the formula is verified for n = 4

To **verify**: substitute, compare with a drawing, table or graph, then write a concluding statement. To **justify**: define every letter, then explain the reason for each number.

Learning

101 Verify that $u_n = 5n - 2$ gives 13 for the 3rd term of the sequence 3, 8, 13, 18

a) Substitute n = 3

b) Write a concluding statement

102 Write verify or justify for each

a) Provide evidence that validates the result

b) Give valid reasons to support an answer

103 A tile pattern has 5, 9, 13, 17 tiles and the formula is $u_n = 4n + 1$

a) Substitute n = 6

b) Continue the table to the 6th term

c) Write a concluding statement

Learning score _____ **/ 21**

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10.1.4 Verify and justify

PRACTISING

Name:

Practising

104 Matchstick squares in a row need 4, 7, 10, 13 matchsticks and the formula is $u_n = 3n + 1$
Verify the formula for pattern 5

105 A student justifies $u_n = 3n + 1$ by writing "it goes up by 3"
Write two things that are missing

106 Justify why the border of an $n \times n$ grid is $n^2 - (n - 2)^2$
Define your letters first

Practising score _____ / 9

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10.1.4 Verify and justify

CHALLENGING

Name:

Challenging

107 In a painted cube, the cubes with exactly 1 painted face are in the middle of each face
Justify the rule $6(n - 2)^2$
Define n and explain the 6 and the $(n - 2)^2$

108 A geometric sequence has $u_1 = 3$ and $r = 2$
a) Write the explicit formula
b) Verify it for u_5 by repeated multiplication
c) Write a concluding statement

Challenging score _____ / 12

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10.1.4 Verify and justify

MASTERY

Name:

Mastery

109 A pattern of dots has 2, 6, 10, 14 dots

a) Find the nth term

b) Verify it for the 7th term

c) Justify your formula using the common difference and the 0th term

110 An arithmetic sequence has $u_1 = 7$ and $d = 5$ and the formula is $u_n = 5n + 2$

Write a full verification and justification: define your letters, substitute a value, check with a second form, give a reason for each number and finish with a concluding statement

Mastery score _____ / 12

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Homework and checking record

HAND THIS PAGE IN WITH YOUR SHEETS

Name:

Your teacher will tell you which sheet and questions to do. Choose your level, set out your work in pen or pencil, then check it with the answers at the back using a **green pen**: circle wrong question numbers, and star ★ any you still cannot solve.

Sheet	Questions	Which ones I did	Checked in green ✔	Teacher
10.1.1a The border problem	Q1–10			
10.1.1b The painted cube	Q11–20			
10.1.1c The painted cube as functions	Q21–30			
10.1.1d What is a sequence?	Q31–40			
10.1.2a Arithmetic sequences	Q41–50			
10.1.2b Tiling patterns and the nth term	Q51–60			
10.1.2c Finding u_1 and d from two terms	Q61–70			
10.1.3a Geometric sequences	Q71–80			
10.1.3b Finding u_1 and r from two terms	Q81–90			
10.1.3c Arithmetic or geometric?	Q91–100			
10.1.4 Verify and justify	Q101–110			

Error analysis

Circle the number of any question you got wrong. Star ★ it if you still cannot solve it after correcting. Then write the questions you need to rework, and what you will do differently:

One thing I understand better now

Score: Parent / guardian signature:

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Answers

CHECK IN GREEN PEN

10.1.1a The border problem

Learning	**1**	b) 16 squares (for example 5 + 5 + 3 + 3, or 4 × 5 − 4)
	2	4 × 18 − 4 = 72 − 4 = 68
	3	a) The top row and bottom row have 7 each, and the two sides each have 5 left between the corners b) 24 c) 4 × 7 − 4 = 24, so they agree
Practising	**4**	a) The 4 corner squares are in two sides each, so they are counted twice b) 44 − 4 = 40
	5	(14 × 14) − (12 × 12) = 196 − 144 = 52. Check: 4 × 14 − 4 = 52
	6	4n = 92, so n = 23: a 23 × 23 grid
Challenging	**7**	n + 2n − 2 + n − 2 = 4n − 4. Collect the n terms (4n) and the numbers (−4)
	8	$(n - 2)^2 = n^2 - 4n + 4$. The minus sign changes every sign inside: $n^2 - n^2 + 4n - 4 = 4n - 4$
Mastery	**9**	Two full rows give 2m and the two sides have n − 2 each, so 2m + 2(n − 2) = 2m + 2n − 4. Check: 10 + 10 + 4 + 4 = 28 and 20 + 12 − 4 = 28
	10	The inside is an 11 × 11 square, so the grid is 13 × 13. Border = 4 × 13 − 4 = 48. Check: 169 − 121 = 48

10.1.1b The painted cube

Learning	**11**	a) 8 × 8 × 8 = 512. b) 8, one for each corner. c) $(8 - 2)^3 = 6^3 = 216$.
	12	n − 2 = 4. a) $6 \times 4^2 = 6 \times 16 = 96$. b) 12 × 4 = 48.
	13	n − 2 = 5. No paint: $5^3 = 125$. 1 face: 6 × 25 = 150. 2 faces: 12 × 5 = 60. 3 faces: 8. Check: 125 + 150 + 60 + 8 = 343 = 7^3.
Practising	**14**	n − 2 = 8. a) 12 × 8 = 96. b) $6 \times 8^2 = 384$. c) 3 faces 8, no paint $8^3 = 512$, so 8 + 96 + 384 + 512 = 1000 = 10^3.
	15	$(n - 2)^3 = 216$, so n − 2 = 6 and n = 8. Check: $6^3 = 216$.
	16	12(n − 2) = 12n − 24. When n = 15: 12 × 13 = 156.
Challenging	**17**	a) $6(n^2 - 4n + 4) = 6n^2 - 24n + 24$. b) 6 × 16 − 24 × 4 + 24 = 96 − 96 + 24 = 24.
	18	$6(n - 2)^2 = 294$, so $(n - 2)^2 = 49$, n − 2 = 7 and n = 9. No paint: $7^3 = 343$.
Mastery	**19**	$(n - 2)^3 = n^3 - 6n^2 + 12n - 8$; $6(n - 2)^2 = 6n^2 - 24n + 24$; 12(n − 2) = 12n − 24. Adding: $n^3 + (-6 + 6)n^2 + (12 - 24 + 12)n + (-8 + 24 - 24 + 8) = n^3$.
	20	Side lengths minus 2: 1, 2, 3. 3 faces: 8. 2 faces: 4(1 + 2 + 3) = 24. 1 face: 2(1×2 + 2×3 + 1×3) = 2 × 11 = 22. No paint: 1 × 2 × 3 = 6. Check: 8 + 24 + 22 + 6 = 60 = 3 × 4 × 5.

10.1.1c The painted cube as functions

Learning	**21**	a) 8 (the corners). b) $(5 - 2)^3 = 3^3 = 27$ (the inside).
	22	a) 12 × 1 = 12. b) 12 × 4 = 48.
	23	0 faces: $4^3 = 64$. 1 face: $6 \times 4^2 = 96$. 2 faces: 12 × 4 = 48. 3 faces: 8. Check: 64 + 96 + 48 + 8 = 216 = 6^3.
Practising	**24**	a) quadratic: a curve called a parabola. b) cubic: a steeper curve. c) linear: a straight line.
	25	0 faces: $7^3 = 343$. 1 face: $6 \times 7^2 = 294$. 2 faces: 12 × 7 = 84. 3 faces: 8. Total: 343 + 294 + 84 + 8 = 729 = 9^3.
	26	12(n − 2) = 108, so n − 2 = 9 and n = 11.
Challenging	**27**	$6(n - 2)^2 = 150$, so $(n - 2)^2 = 25$ and n − 2 = 5, so n = 7. Inside cubes: $5^3 = 125$.
	28	n = 6: inside $4^3 = 64$ and edges 12 × 4 = 48, so 64 is more. n = 5: inside 27 and edges 36, so not yet. Algebra: $(n - 2)^3 > 12(n - 2)$ gives $(n - 2)^2 > 12$, so $n - 2 \geq 4$ and n = 6.
Mastery	**29**	n is the number of small cubes along an edge, so it is a whole number, and a 1 × 1 × 1 cube has all six faces painted so the pattern breaks. For n less than 2 the formulae give negative or meaningless values. A cube of side 3.5 does not exist, so points between whole numbers mean nothing: the graph is a set of separate points.
	30	For n = 3: cubic 1, quadratic 6, linear 12, so the cubic is the smallest. They are equal at n = 8 (216 for both cubic and quadratic). From n = 9 the cubic is largest: 343 against 294 and 84. So the cubic wins in the long run but not for small n.

10.1.1d What is a sequence?

Learning	**31**	a) 6. b) 21. c) u_3 is the 3rd term, so $u_3 = 16$.
	32	a) The terms go up by 4: 21, 25. b) The terms go down by 9: 54, 45. c) Each term doubles: 48, 96.
	33	12, 7, 2, −3, −8.
Practising	**34**	a) Put n = 1, 2, 3, 4, 5 into 3n + 4: 7, 10, 13, 16, 19. b) 8, 6, 4, 2, 0.
	35	$u_2 = 4 + 7 = 11$, $u_3 = 18$, $u_4 = 25$, $u_5 = 32$.
	36	a) $u_8 = 5(8) - 2 = 38$. b) $5n - 2 = 43$, so $5n = 45$ and $n = 9$. The term 43 is in position 9.
Challenging	**37**	Yes, both rules fit the first three terms. Sam adds 3 each time. Mia repeats 3, 6, 9. A third way: add 3, then 4, then 5, ... giving 3, 6, 9, 13, 18. A few terms do not decide the sequence. We need a rule or a real context.
	38	a) Explicit: $u_5 = 25 + 1 = 26$. b) Recursive: 2, 5, 14, 41, 122, so $u_5 = 122$. c) Explicit: $u_5 = 20 - 15 = 5$.
Mastery	**39**	2, 3, 5, 8, 13, 21, 34, 55. Each term is the sum of the two terms before it.
	40	A recursive formula gives each term from the term before and needs the first term. An explicit formula gives any term from its position n. For u_{50} the explicit formula is quicker, because you only put n = 50 into it. The recursive formula needs all 49 terms before.

10.1.2a Arithmetic sequences

Learning	**41**	a) Yes, d = 5. b) Yes, d = −4. c) No, the differences are 2, 4, 8 and are not equal. d) No, the differences are 2, 3, 4 and are not equal.
	42	a) 6, 9, 12, 15. b) 20, 15, 10, 5. c) 1, 8, 15, 22.
	43	a) $u_1 = 6 \times 1 + 1 = 7$, $u_2 = 13$, $u_3 = 19$. b) $u_{10} = 6 \times 10 + 1 = 61$.
Practising	**44**	a) $u_1 = 9$ and d = 4. b) $u_n = 9 + (n - 1) \times 4 = 9 + 4n - 4 = 4n + 5$. c) $u_{30} = 4 \times 30 + 5 = 125$.
	45	a) $u_n = u_{n-1} - 6$ with $u_1 = 50$. b) $u_n = 50 + (n - 1) \times (-6) = 56 - 6n$. c) $u_{15} = 56 - 90 = -34$.
	46	a) $3n - 1 = 100$ gives $n = 101 \div 3$, which is not a whole number, so 100 is not a term. b) $3n - 1 = 101$ gives $n = 34$, a whole number, so 101 is the 34th term.
Challenging	**47**	a) $u_1 = 5$ and d = 3, so $u_n = 5 + (n - 1) \times 3 = 3n + 2$. b) $u_{40} = 3 \times 40 + 2 = 122$. c) $3n + 2 = 98$ gives $3n = 96$ and $n = 32$, so pattern 32.
	48	a) $u_3 = u_1 + 2d$, so $14 = u_1 - 6$ and $u_1 = 20$. b) $u_{10} = 20 + 9 \times (-3) = -7$.
Mastery	**49**	a) 5, 20, 45. b) The differences are 20 − 5 = 15 and 45 − 20 = 25. They are not equal, so the sequence is not arithmetic.
	50	a) $u_{n+1} = 4(n + 1) - 3 = 4n + 1$. b) $u_{n+1} - u_n = (4n + 1) - (4n - 3) = 4$, whatever the value of n. c) The difference between consecutive terms is always 4, so d = 4. This is a proof for every n, not just a few terms.

10.1.2b Tiling patterns and the nth term

Learning	**51**	a) d = 4. b) 11 + 4 = 15 tiles. c) For example pattern 4 adds one more tile to the end of each of 4 arms, 4 new tiles, so 15 tiles. Conclusion: prediction verified.
	52	Multiples of 3: 3, 6, 9, 12, 15. Adjust: +2 each time. $u_n = 3n + 2$.
	53	d = 10, multiples 10, 20, 30, 40, so add 2. $u_n = 10n + 2$. $u_{15} = 10(15) + 2 = 152$.
Practising	**54**	d = −3, multiples −3, −6, −9, −12, so add 21. $u_n = 21 - 3n$. $u_{10} = 21 - 30 = -9$.
	55	a) d = 5, multiples 5, 10, 15, 20, so subtract 1. $u_n = 5n - 1$. b) $u_{40} = 5(40) - 1 = 199$.
	56	$u_1 = 6(1) - 5 = 1$ and $u_3 = 6(3) - 5 = 13$, and $u_4 = 6(4) - 5 = 19$. These match the sequence. Conclusion: the rule gives the same terms as the sequence, so $u_n = 6n - 5$ is verified.
Challenging	**57**	a) $3n + 5 = 83$ gives $3n = 78$ and $n = 26$, a whole number, so 83 is the 26th term. b) $3n + 5 = 100$ gives $3n = 95$ and $n = 31.67$, not a whole number, so 100 is not a term.
	58	a) The points go down 2 each time, so the gradient is −2. b) The 0th term is 9 + 2 = 11, so the y-intercept is 11. c) $u_n = 11 - 2n$.
Mastery	**59**	a) 5, 7, 9: not constant, so not arithmetic. b) 2, 2: constant, so the sequence is quadratic. c) $u_n = n(n + 2) = n^2 + 2n$. Pattern 10 has $10 \times 12 = 120$ tiles.
	60	a) $u_0 = -12$. b) $5n - 12 > 0$ means $n > 2.4$, so $n = 3$ and $u_3 = 3$. c) n must be a whole number, so there is no 2.5th term and the points in between have no meaning.

10.1.2c Finding u_1 and d from two terms

Learning	**61**	6 − 2 = 4 jumps.
	62	a) 5 − 2 = 3 jumps. b) 3d = 19 − 7 = 12, so d = 4.
	63	$u_4 = 2 + 3 \times 6 = 20$. $u_9 = 2 + 8 \times 6 = 50$.
Practising	**64**	4 jumps, 4d = 20, d = 5. $u_2 = u_1 + d$ so $10 = u_1 + 5$ and $u_1 = 5$.
	65	4 jumps, 4d = −16, d = −4. $u_3 = u_1 + 2d$ so $4 = u_1 - 8$ and $u_1 = 12$.
	66	6 jumps, 6d = 36, d = 6. $u_4 = u_1 + 3d$ so $23 = u_1 + 18$ and $u_1 = 5$. $u_{15} = 5 + 14 \times 6 = 89$.
Challenging	**67**	d = 5. a) $6 + (n - 1) \times 5 = 106$, n − 1 = 20, n = 21. b) (150 − 6) ÷ 5 = 28.8, not a whole number, so 150 is not a term.
	68	7 jumps, 7d = −28, d = −4, $u_1 = 8 + 16 = 24$. $u_n = 24 + (n - 1)(-4) = 28 - 4n$. $u_8 = 24 - 28 = -4$ and $28 - 32 = -4$, so verified.
Mastery	**69**	$u_2 + u_6 = 2u_1 + 6d = 34$ so $u_1 + 3d = 17$. $u_1 + 8d = 41$. Subtract: 5d = 24, d = 4.8, $u_1 = 2.6$. Check $u_2 + u_6 = 7.4 + 26.6 = 34$.
	70	$u_1 + 9d = 3(u_1 + 3d)$ gives $0 = 2u_1$, so $u_1 = 0$. $u_7 = 6d = 21$ so d = 3.5. Check $u_4 = 10.5$ and $u_{10} = 31.5$.

10.1.3a Geometric sequences

Learning	**71**	a) 108, 324, 972 (× 3) b) 4, 0.8, 0.16 (× $\frac{1}{5}$) c) −16, 32, −64 (× −2)
	72	a) 2 b) $\frac{1}{3}$ c) −3 d) 1.5
	73	a) arithmetic, d = 5 b) geometric, r = 2 c) neither (differences 3, 5, 7 and ratios 4, 2.25) d) geometric, r = $\frac{1}{2}$
Practising	**74**	a) 5, 15, 45, 135 b) $u_n = u_{n-1} \times 3$, $u_1 = 5$
	75	a) $4 \times 2^5 = 4 \times 32 = 128$ b) $4 \times 2^7 = 4 \times 128 = 512$
	76	a) $3 \times 2^6 = 3 \times 64 = 192$ b) r = $\frac{1}{3}$, so $u_7 = 729 \times (\frac{1}{3})^6 = 729 \div 729 = 1$
Challenging	**77**	a) $u_4 = 2 \times r^3 = 54$, so $r^3 = 27$ and r = 3 b) $u_6 = 2 \times 3^5 = 2 \times 243 = 486$
	78	From u_2 to u_4 is two steps of × r, so $r^2 = 80 \div 20 = 4$, and r = 2 or r = −2. If r = 2 then $u_1 = 20 \div 2 = 10$. If r = −2 then $u_1 = 20 \div (-2) = -10$
Mastery	**79**	a) 100% + 4% = 104% = 1.04 b) $u_3 = 520 \times 1.04^2 = 520 \times 1.0816 = 562.432$, so £562.43
	80	a) The 3rd term: 20 is bigger than 15 b) A adds 5 each time, but B multiplies by 2, so the gap between them keeps growing

10.1.3b Finding u_1 and r from two terms

Learning	**81**	4, 12, 36, 108. Multiply by 3 each time
	82	One step, so r = 40 ÷ 8 = 5
	83	7 − 3 = 4 steps
Practising	**84**	Three steps, $r^3 = 54 \div 2 = 27$, so r = 3
	85	Three steps, $r^3 = 56 \div 7 = 8$, r = 2. $u_1 = 7 \div 2 = 3.5$
	86	Three steps, $r^3 = 540 \div 20 = 27$, r = 3. $u_1 = 20 \div 9 = 20/9$. $u_7 = 540 \times 3 = 1620$
Challenging	**87**	Two steps, $r^2 = 2 \div 18 = 1/9$, so r = 1/3 or r = −1/3. $u_1 = 18 \div r = 54$ or −54
	88	$r^3 = 8$, r = 2, $u_1 = 3$, $u_n = 3 \times 2^{n-1}$. $u_8 = 384$ and $u_9 = 768$, so u_9 is the first term over 500
Mastery	**89**	$x \div 5 = 45 \div x$, so $x^2 = 225$ and x = 15 or x = −15
	90	Four steps, $r^4 = 1296 \div 16 = 81$, so r = 3 or r = −3. $u_1 = 16 \div 9 = 16/9$. With r = 3: 16/9, 16/3, 16. With r = −3: 16/9, −16/3, 16. Verify: $u_7 = 16 \times 3^4 = 1296$ ✓

10.1.3c Arithmetic or geometric?

Learning	**91**	a) arithmetic, d = 5 (11 − 6 = 5, 16 − 11 = 5, 21 − 16 = 5) b) geometric, r = 2 (10 ÷ 5 = 2, 20 ÷ 10 = 2, 40 ÷ 20 = 2) c) neither (differences 1, 2, 3 are not equal and ratios 2, 2, 1.75 are not equal)
	92	a) r = 2, so the next terms are 112 and 224 b) d = −7, so the next terms are 62 and 55
	93	$u_1 = 5$ and d = 3, so $u_n = 5 + (n - 1) \times 3 = 3n + 2$. Check: n = 4 gives 14
Practising	**94**	a) r = 12 ÷ 3 = 4 b) $u_n = 3 \times 4^{n-1}$ c) $u_6 = 3 \times 4^5 = 3 \times 1024 = 3072$
	95	a) $u_n = u_{n-1} - 3$, $u_1 = 20$ b) $u_n = u_{n-1} \times 5$, $u_1 = 2$
	96	a) Plan A: £6, £12, £18, £24, £30, £36. Plan B: 2p, 4p, 8p, 16p, 32p, 64p b) Plan A pays £36 and Plan B pays only 64p, so Plan A is greater on day 6 c) Plan A is arithmetic so linear growth, Plan B is geometric so exponential growth
Challenging	**97**	Plan B on day n pays $2 \times 2^{n-1}$ pence = 2^n pence. Day 10: 1024p = £10.24. Day 12: 4096p = £40.96. Plan A pays 6n pounds: day 10 £60, day 12 £72. Plan B on day 15: 32768p = £327.68 against £90. Day 13: £81.92 against £78, so day 13 is the first day Plan B pays more
	98	$u_2 = 15$, $u_3 = 45$, $u_4 = 135$. The differences are 10, 30, 90, which are not equal. Each difference is three times the one before, because the terms are multiplied by 3 so the gaps are multiplied by 3 as well
Mastery	**99**	a) 24, 48 and 96 b) 40 days is 10 periods of 4 days, so $12 \times 2^{10} = 12 \times 1024 = 12\,288$ cases c) 12 + 12 × 10 = 132 cases. Doubling gives about 93 times more cases after 40 days: multiplying beats adding
	100	$u_1 = 2$ and r = 3 so $u_n = 2 \times 3^{n-1}$. Check n = 3: 2 × 9 = 18. We need $2 \times 3^{n-1} > 1\,000\,000$, so $3^{n-1} > 500\,000$. $3^{11} = 177\,147$ and $3^{12} = 531\,441$, so n − 1 = 12 and n = 13. The 13th term is 2 × 531 441 = 1 062 882

10.1.4 Verify and justify

Learning	**101**	a) 5 × 3 − 2 = 15 − 2 = 13 b) The formula gives 13 and the 3rd term of the sequence is 13, so the formula is verified for n = 3
	102	a) verify b) justify
	103	a) 4 × 6 + 1 = 25 b) 5, 9, 13, 17, 21, 25 c) The formula gives 25 and the table gives 25, so the formula is verified for n = 6
Practising	**104**	3 × 5 + 1 = 16. Continuing the table gives 4, 7, 10, 13, 16, which matches, so the formula is verified for pattern 5
	105	Any two of: n and u_n are not defined, the + 1 is not explained (4 + 3(n − 1) = 3n + 1), there is no link to the drawing
	106	Let n be the side length of the grid. The whole grid has n^2 squares. Removing the border leaves a square that is n − 2 wide, so $(n - 2)^2$ squares are not on the border. The border is the difference, $n^2 - (n - 2)^2$
Challenging	**107**	Let n be the side length of the large cube. The middle of each face is an (n − 2) by (n − 2) square of small cubes because the outer ring touches an edge. That gives $(n - 2)^2$ on one face and there are 6 faces, so $6(n - 2)^2$
	108	a) $u_n = 3 \times 2^{n-1}$ b) $u_5 = 3 \times 2^4 = 48$ and 3, 6, 12, 24, 48 gives 48 c) Both methods give 48, so the formula is verified for n = 5
Mastery	**109**	a) $u_n = 4n - 2$ b) $u_7 = 4 \times 7 - 2 = 26$ and continuing 18, 22, 26 gives 26, so verified c) Let n be the position and u_n the number of dots. The common difference is 4, so the formula starts 4n. 4n gives 4, 8, 12, 16 which is 2 too big each time, so subtract 2. This matches the 0th term, 2 − 4 = −2
	110	Let n be the position and u_n the nth term. Verify: $u_4 = 5 \times 4 + 2 = 22$ and 7, 12, 17, 22 gives 22, so the formula is verified for n = 4. Justify: the 5 is the common difference d. The + 2 comes from $u_1 - d = 7 - 5 = 2$, the 0th term. Conclusion: the formula $u_n = 5n + 2$ is correct because it gives $u_1 = 7$ and adds 5 for each extra term